Advantage of Base-Line Redundancy in Sparse Apertures

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Advantage of Base-Line Redundancy in Sparse Apertures

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Abstract

A general argument is presented to explain the dependence of observation-time T on sparseness f in observations with sparse apertures. For a system using a single observation, $T \sim f^{-3}$, while for a system using multiple observations, $T \sim f^{-2}$. When multiple observations are used, the optimum strategy is to maximize the redundancy of base-lines within each observation but to minimize the redundancy of base-lines between observations.

We consider an imaging system designed to achieve optimum signal-to-noise ratio when observing a scene with uniform bright-field background. The resolution and field-of-view are chosen to be equivalent to the diffraction-limited resolution and field-of-view of a filled aperture telescope of given size, but the actual system uses a sparse-aperture telescope covering a fraction f of the filled aperture. We suppose that the number of pixels in the diffraction-limited image is N. The filled aperture can usefully be considered as consisting of N patches and the sparse aperture as consisting of fN patches. Each Fourier component of the image brightness is the result of interference of light reflected from two patches of the aperture, with a base-line equal to the vector separation of the two patches. There is a one-to-one correspondence between Fourier components of the image and base-lines of the aperture. We use the index $k = 1, 2, ..., N$ to refer to both Fourier components and base-lines. The same label k refers also to the Fourier components of the brightness of the scene.

In order to obtain full resolution of the scene, it is essential to use apertures including all base-lines. To obtain uniform resolution of details, it is desirable to cover all base-lines as uniformly as possible. But there are two ways to achieve uniform coverage of base-lines. Method 1 uses a single sparse aperture covering all base-lines. Method 2 divides the observation of a single scene into n intervals of time, using n different sparse apertures for the observations. Using Method 2, each individual aperture does not need to cover all base-lines. It is sufficient if the apertures collectively cover all base-lines. In Method 2, the number n of observations is to be chosen to optimize the signal-to-noise ratio of the image obtained by combining n observations within a total duration T. Method 1 is the special case of Method 2 when $n = 1$, so there is only a single observation of duration T. The n observations are labeled by the index $m = 1, 2, ..., n$.

Let R_{km} be the redundancy of base-line k in the aperture used for observation m, i.e., the number of pairs of patches in that antenna separated by the vector k. For each observation, the sum of the redundancies is equal to the number of pairs of patches,

$$\textstyle\sum_k R_{km} = \frac{1}{2} f^2 N^2 \,, \quad m = 1, ..., n \,. \tag{1}$$

Now consider the signal S_{km} and noise N_{km} in the observation m of Fourier component k of the scene

$$S_{km} = (T/n) R_{km} B_k \,, \tag{2}$$

$$N_{km} = [(T/n) R_{0m} U]^{1/2} = [(T/n) f N U]^{1/2} \,, \tag{3}$$

where B_k is the brightness of Fourier component k of the scene, U is the uniform background brightness, and $R_{0m} = fN$ is the number of patches in the antenna for each observation.

The best estimate of the scene brightness B_k is a linear combination of the signals S_{km},

$$E_k = (n/T) \textstyle\sum_m (R_{km}/V_k) S_{km} \,, \tag{4}$$

$$V_k = \textstyle\sum_m R_{km}^2 \,, \tag{5}$$

each weighted in proportion to its signal-to-noise ratio. E_k is Fourier component k of the best image that can be reconstructed from the n observations. The estimate E_k measures B_k with noise

$$W_k = [n f N U / T V_k]^{1/2} \,, \tag{6}$$

according to (3) and (4). To obtain an image with uniformly low noise, all the W_k should be small.

We take as figure-of-merit for the system as a whole the root-mean-square signal-to-noise ratio of all the Fourier components of the image,

$$\begin{aligned} F &= [\textstyle\sum_k E_k^2 / \sum_k W_k^2]^{1/2} \\ &= \left[\frac{TB^2}{nfU}\right]^{1/2} \left[\textstyle\sum_k \frac{1}{V_k}\right]^{-1/2}, \end{aligned} \tag{7}$$

where

$$B = [\tfrac{1}{N} \textstyle\sum_k B_k^2]^{1/2} \tag{8}$$

is the root-mean-square signal. To make (7) large, all the sums (5) should be uniformly as large as possible. Let

$$Y_k = \textstyle\sum_m R_{km} \tag{9}$$

be the total redundancy of base-line k in all the n observations. The sum of the Y_k is fixed by (1),

$$\textstyle\sum_k Y_k = \frac{1}{2} n f^2 N^2 . \tag{10}$$

To make (7) large, we should first distribute the total redundancy as equally as possible over the base-lines, so that all the Y_k are roughly equal to the average

$$Y_k \sim Y = \tfrac{1}{2} n f^2 N , \tag{11}$$

and then distribute the redundancy of each base-line as unequally as possible over the observations, so that one or two terms R_{km}^2 dominate in each sum (5). So far as possible, each base-line should be used in only one of the n observations. The system is optimized when the redundancies are minimized between observations and maximized within observations. If this is done, each V_k is approximately equal to the square of the corresponding Y_k,

$$V_k \sim Y_k^2 \sim \tfrac{1}{4} n^2 f^4 N^2 \tag{12}$$

according to (11). But the maximum redundancy of any base-line in any observation is fN, so that

$$V_k \leq fNY_k \sim \tfrac{1}{2} n f^3 N^2 \,, \tag{13}$$

by (11). Comparing (12) with (13),

$$fn \leq 2 \,. \tag{14}$$

The maximum number of observations that can be combined efficiently is

$$n_{\max} = 2f^{-1} \,. \tag{15}$$

For geometrical reasons, it is not possible to use all base-lines in exactly one observation each. The shortest base-lines will necessarily appear in all the observations. But the majority of base-lines are not short, and it should be possible to arrange for the long ones to appear in only one or two observations. The approximate equality (12) will then hold for the majority of base-lines.

When (12) holds, the figure-of-merit of the system according to (7) is

$$F = [(B^2 N/4U) n f^3 T]^{1/2} \,. \tag{16}$$

For a given scene brightness B, background brightness U, and signal-to-noise ratio F, the total observation-time T varies with n and f according to

$$T \sim (nf^3)^{-1} \,. \tag{17}$$

This means that for a system using only one observation (Method 1),

$$T \sim f^{-3}, \tag{18}$$

while for a system using the optimum number (15) of observations (Method 2),

$$T \sim f^{-2}. \tag{19}$$

In particular, a system using a single observation with a ring-shaped or Y-shaped antenna will have $T \sim f^{-3}$, while a system with a rotating rectangular antenna observing in $(2f^{-1})$ different orientations will have $T \sim f^{-2}$. The rectangular antenna is in principle more efficient when f is very small. For practical values of f, the theoretical advantage of multiple observations may be insignificant, and a system using a single observation may be preferred for other reasons.

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